Reconstruction of 3D Lines from a Single Axial Catadioptric Image Using Cross-Ratio*

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Abstract

It has been shown, in previous work, that the 3D position of a line can be reconstructed from a single image in vision systems that do not possess a single viewpoint. We present a new method that, in a non-central axial catadioptric system, can achieve line spacial reconstruction from 3 or more image points, given the distance ratio of 3 points in the line (a fair assumption in, for example, structured environment with repetitive architectural features). We use cross-ratio as an invariant to constrain the line localization and perform the reconstruction from a set of image points through non-linear optimization. Experimental results are presented.

1. Introduction

Catadioptric vision systems use a combination of cameras and mirrors to acquire images. A particular class of systems, central catadioptric cameras, allow for a single-viewpoint projection model [1]. In general, however, a catadioptric camera is non-central [10], and the viewing rays do not intersect at a single point.

The multi-viewpoint characteristic of non-central cameras has been explored to achieve spacial localization of lines from a single image [3–6, 8]. In non-central systems, the viewing rays corresponding to a line in the image form a non-planar surface that, in a general case, allows only for a finite number of possible space lines transversal to the viewing rays.

In [3], Caglioti and Gasparini discussed constraints on system geometry and line positioning that enable the line to be univocally located. They also presented an algorithm to reconstruct a line from 4 image points, assuming an axial-symmetrical system. In a closely related paper [8], Lanman et al. shown how to reconstructed a line spacial position using Singular-Value-Decomposition of a matrix of Plücker coordinates of 4 viewing rays. The constraint of placing the camera on the mirror axis was relaxed in [4], and the conditions under which lines can be reconstructed were analyzed in this more general system geometry. In [5], Caglioti et al. proposed two new methods for line localization. One method first tried to identify two planar viewing rays (whose existence is not guaranteed for a line in general position), and then use two more rays to provide a simple geometric solution for the reconstruction. The other method relied on a constrained non-linear optimization whose error function was based on a bilinear operator of Plücker vectors. In [6], the image of a space line was used to provide constraints to the calibration of an off-axis catadioptric camera.

In this paper we propose a new algorithm to reconstruct the spacial localization of lines from a single image, acquired from a non-central axial catadioptric system. The system is composed of a rotationally symmetric mirror and a pinhole camera with its optical center placed on the mirror axis. We assume that the system is calibrated, so that each image point corresponds to a known viewing ray in space. Our algorithm uses the images of 3 points on a space line (the previously cited methods use 4 points), and requires the knowledge of the ratio of distances between those 3 points. In structured environments, the distance ratio can be determined, for example, from repetitive features in the floor, walls or ceiling, like windows, light fixtures, tiles, etc.

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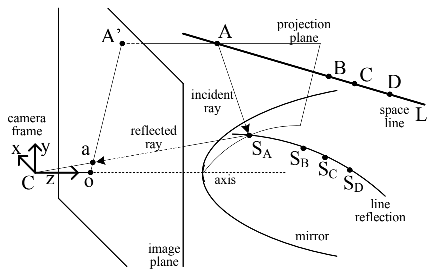

Figure 1. The axial catadioptric geometry.

By using the cross-ratio as an invariant in our image formation geometry, we show how to obtain a constraint on the line spacial direction, and determine its localization through non-linear optimization of a single variable function. All the points on the image contour of the line can be integrated in the non-linear optimization to reduce the effect of noise.

2. Cross-ratio as an invariant in the axial catadioptric geometry

In this section we discuss the system geometry and show that the cross-ratio can be used as an invariant between scene and image points. This property was first noted by Wu and Hu in [11], that used it in the calibration of central catadioptric systems. We show how it can be applied to our model and used to constrain the direction of the space line.

As previously mentioned, we consider an non-central axial catadioptric system. Additionally, we assume that the camera is aligned with the mirror, i.e., the camera's principal axis coincides with the symmetry axis of the mirror. A pre-rotation can be performed to align the camera frame with the mirror axis by applying a homographic point transformation to the image (called a *conjugate rotation* [7]).

Consider Fig. 1. Let $\mathbf{C}$ be the camera's optical center and $\mathbf{o}$ the principal point of the image (image center). Let $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ be four 3D points belonging to a space line. Point $\mathbf{a}$ is the reflected image of $\mathbf{A}$. Point $\mathbf{S}_A$ is the reflection point on the surface of the mirror. From the laws of reflection, we know that the incident ray, the reflected ray and the mirror surface normal at point $\mathbf{S}_A$ must belong to the same plane. We refer to this plane as the *projection plane* of point $\mathbf{A}$. Note that, given the mirror's rotational symmetry, every *projection plane* must contain the mirror axis.

Since the camera principal axis (z-axis) is aligned with the mirror axis, the orthographic projection of point $\mathbf{A}$ in the image plane, denoted by $\mathbf{A}'$, is also on the same *projection plane*. Thus, in the image, the principal point $\mathbf{o}$, the orthographic projection $\mathbf{A}'$, and the reflected image $\mathbf{a}$ are collinear. Fig. 2(a) shows this relation for images of the four image points. It is evident that the cross-ratio of the four points $\mathbf{A}'$, $\mathbf{B}'$, $\mathbf{C}'$ and $\mathbf{D}'$ is equal to the cross-ratio between the four concurrent lines passing through $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ and the vertex $\mathbf{o}$ (c.f. [9]). Furthermore, since the cross-ratio is invariant under an orthographic projection we can write ($\{\cdot\}$ denotes cross-ratio)

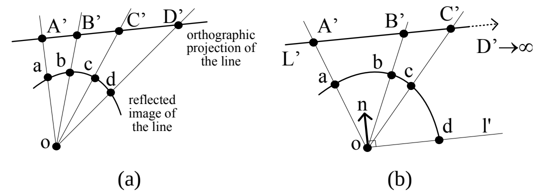

Figure 2. The cross-ratio in the image.

$$\{\mathbf{o};\mathbf{abcd}\} = \{\mathbf{A}'\mathbf{B}'\mathbf{C}'\mathbf{D}'\} = \{\mathbf{ABCD}\}\,. \qquad (1)$$

3. Reconstructing a space line

In this section we present our reconstruction method.

Constraint on the line direction

Consider Fig. 2(b), where $\mathbf{L}'$ is the orthographic projection of the space line and $\mathbf{l}'$ is the image line joining points $\mathbf{o}$ and $\mathbf{d}$. If point $\mathbf{D}$ is considered to be at infinity, $\mathbf{D} \to \infty$, then $\mathbf{l}'$ becomes parallel to $\mathbf{L}'$, and its direction can be obtained from equation 1, given the knowledge of the distance ratio between three (finite) points of the space line, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

Let $\mathbf{n} = \begin{bmatrix} n_x & n_y & 0 \end{bmatrix}^\mathsf{T}$ be a 3D vector that belongs to the image plane and is orthogonal to line $\mathbf{l}'$, pointing in the direction of the orthographic projection of the space line (w.r.t. the image center $\mathbf{o}$). Also, consider this vector to be normalized to unit length, $\parallel \mathbf{n} \parallel = 1$.

Vector $\mathbf{n}$ can be seen as the normal vector to a family of planes that are perpendicular to the image plane and are parallel to the space line $\mathbf{L}$. This family of planes can be parameterized by $\Pi(\alpha) \sim \begin{bmatrix} n_x & n_y & 0 & -\alpha \end{bmatrix}^\mathsf{T}$, where $\alpha > 0$ is a scalar. Note that a 3D point $\mathbf{X}$ belongs to plane $\Pi(\alpha)$ *iff* $\begin{bmatrix} \mathbf{X}^\mathsf{T} & 1 \end{bmatrix} \Pi(\alpha) = 0$.

3D reconstruction

The spacial line must belong to the surface of the viewing rays back-projected from the line image, and, at the same time, to a plane in the family of planes $\Pi(\alpha)$. Assuming general position, the curved surface of the viewing rays contains two lines transversal to the viewing rays [3]. One solution is the mirror axis, as all rays pass through it, and corresponds to plane $\Pi(\alpha = 0)$. The other solution corresponds to the spacial line itself. The problem is, thus, reduced to finding the value of $\alpha > 0$ that produces a line as the result of intersecting plane $\Pi(\alpha)$ with the surface of the viewing rays.

Consider N viewing rays obtained from back-projecting image points on the line. Each ray is defined by the reflection point on the surface of the mirror $\mathbf{S}_i$ and by a direction vector $\mathbf{R}_i$. The subscript i denotes the i-th ray. The intersection point between a viewing ray and plane $\Pi(\alpha)$ is given by

$$\mathbf{X}_i(\alpha) = \mathbf{S}_i + \left(\frac{\alpha - \mathbf{n}^\mathsf{T}\mathbf{S}_i}{\mathbf{n}^\mathsf{T}\mathbf{R}_i} \right) \mathbf{R}_i \ . \tag{2}$$

Now, consider a matrix Q that is constructed by stacking the set of N intersection points in the following manner:

$$\mathsf{Q}(\alpha) = \begin{bmatrix} \mathbf{X}_1(\alpha)^\mathsf{T} & 1 \\ \multicolumn{2}{c}{\cdots} \\ \mathbf{X}_N(\alpha)^\mathsf{T} & 1 \end{bmatrix}$$

The right null space of Q defines a plane containing all the intersection points $\mathbf{X}_i(\alpha)$, $i = 1, .., N$. This null space always exists because the points belong to plane $\Pi(\alpha)$. However, if all the points are collinear, Q will have a 2-dimensional null space that spans a pencil of planes (with the line as the axis of the pencil) [7].

Assuming the presence of noise, the spacial line localization can be estimated from the following procedure: Consider function $f(\cdot)$ that returns the second smallest singular value resulting from a Single-Value-Decomposition (SVD) of a matrix. The value of α that produces the "best" set of intersection points $\mathbf{X}_i(\alpha)$, $i = 1, .., N$, can be obtained by applying non-linear optimization methods (e.g. Levenberg-Marquardt) to

$$\min_{\alpha} \ f\left(\mathsf{Q}(\alpha)\right) \ . \tag{3}$$

The space line $\mathbf{L}$ is obtained by fitting the set of intersection points.

Using concurrency and perpendicularity to improve the reconstruction

In some situations, additional information regarding the scene may be available, which can be used to improve the reconstruction accuracy and immunity to noise. We outline the procedure that can be used when two distinct lines have a common intersection point visible in the image and are known to be perpendicular.

Let $\mathbf{X}_i(\alpha)$, $i = 1, .., N$ be the set of viewing rays of one line, and $\mathbf{Y}_i(\beta)$, $i = 1, .., M$ the set of the other line. Each set is parameterized by a different scalar, α and β, because each line is associated to different family of planes that constrain its direction in space. Let $\mathbf{C}$ denote the viewing ray corresponding to the concurrency point identified in the image. Since $\mathbf{C}$ belongs to both lines, we have that $\mathbf{C}(\alpha) = \mathbf{C}(\beta)$ and, substituting in equation 2, it is straightforward to obtain β as a function of α, so that, once again, the reconstruction problem reduces to optimizing a function in a single variable. In this case, the objective function f (equation 3) should return a measure of orthogonality, e.g. the inner product between the direction vectors of both lines.

4. Experimental Results

We now present some experimental results to validate our reconstruction algorithm.

Fig. 3(a) shows the image obtained using a spherical mirror (30cm radius) that is reflecting planar checkered patterns. Points belonging to two distinct lines were identified in the image: 8 points in line 1 (green '$\bullet$'), and 7 points in line 2 (blue '$\diamond$'). In each line, three points in the set were selected and used to calculate the distance ratio (marked with '$+$').

We calibrated the system using the following procedure: First, the camera's internal parameters were estimated using standard methods [2]. An auxiliary grid pattern (visible on the upper left corner of Fig. 3(a)), with a known pose w.r.t. the mirror, was used to obtain the camera/mirror transformation directly from the image (again using [2]). Finally, the transformation between the mirror and the remaining checkered patterns was recovered from another perspective image, external to the scene.

Table 1 summarizes the reconstruction results for each line using all the marked points and using only the 3 points with known distance ratio (minimum number of points in our method). The results obtained by applying the 4 points method in [3] to our setup are provided for comparison. Also shown is the result of reconstructing both lines simultaneously using the knowledge that they are concurrent and perpendicular in space. The results are quantified by a distance error d_{err}, obtained from the average distance between the end points of the real and reconstructed line segments, and an angle error γ_{err}, the angle between the real and reconstructed lines. Also shown is the percentage distance error, which is the ratio between d_{err} and the average of the distances between the end points of the real line and the center of the camera.

In comparison to the method in [3], our algorithm produced better results. The simultaneous reconstruction of both lines performed better than each of the individual results, a natural consequence of using more

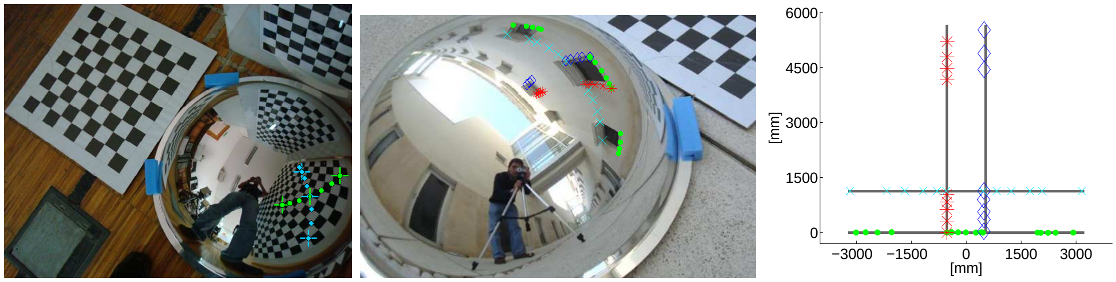

Figure 3. Test images and reconstruction results. Fig.(a), (b) and (c), from left to right.

	4 points (method in [3])			3 points (our method)			all points (our method)		
line 1	346	39.6%	23.7°	31	3.6%	1.2°	53	6.0%	1.0°
line 2	259	29.8%	24.3°	190	21.8%	2.4°	106	12.2%	2.0°
both	–	–	–	–	–	–	43	5.3%	1.2°

Table 1. Experimental results. Each cell shows: d_{err}[mm]; $d_{err}[\%]$; $\gamma_{err}[°]$

information about the scene. As observed in [8], the reconstruction accuracy is very sensitive to noise in the system's calibration and in image point identification.

Fig. 3(b) shows a test image of an outdoors scene with the facade of a building (the image is slightly cropped to provide the reader more detail of the reflected image). Points belonging to 4 lines were marked using visible features of the windows and wall (line 1: green '•'; line 2: cyan '×'; line 3: red '∗'; line 4: blue '⋄';). First, each line was reconstructed individually. Unlike the previous experiment, the ground truth spacial position of the lines (w.r.t. the camera frame) was not available, so we compared the length of the reconstructed and real line segments. The line segments were reconstructed within 30% to 50% of the real length. Next, we paired line 1 with line 3, and line 2 with line 4, and reconstructed each pair using the fact that the lines are concurrent and perpendicular. This time, the length of each line was recovered to within 10% of the real value (worst case). Furthermore, and although each pair was reconstructed independently, all the recovered lines were approximately coplanar. Fig. 3(c) shows the reconstructed line points projected to the recovered "wall" plane (obtained from a least-squares fitting), overlaid on the ground truth line segments (black lines). The distances between the reconstructed 3D points and the estimated plane had an RMS value of 75mm.

The outdoors scene proved more challenging and produced poor results on the recovery of individual lines, although the reconstruction of lines pairs performed very well.

5. Conclusions

We presented a new method for spacial reconstruction of lines from a single image of a non-central axial catadioptric systems. We use knowledge about the scene structure, namely the distance ratio of 3 points, to constrain the line 3D position and facilitate the reconstruction. Our experimental results show that, although the reconstruction can be very sensitive to noise, good results can be achieved even with outdoors scenes.

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